

Course title: **Calculus**  
 Course No: Math Ed. 417  
 Nature of the course: Theory  
 Level: B.Ed. Four Year  
 Year: First

Full marks: 100  
 Pass marks: 35  
 Periods per week: 6  
 Total periods: 150  
 Time per period: 55 minutes

### 1. Course Description

This course is designed to acquaint students with the fundamental principles, techniques and applications of differential calculus, integral calculus and differential equations. It aims to help them to build foundation for higher studies in mathematics.

### 2. General Objectives

The general objectives of this course are as follows:

- To provide the students with an in-depth understanding of the techniques, principles and applications of the differential calculus.
- To make the students familiar with the expansion of various functions in finite and infinite series.
- To enable the students in applying the differential calculus to solve the problems of other branches of mathematics and the problems of maxima and minima.
- To impart knowledge to the students in using differential calculus to study the properties of tangent and normal of a curve, rate of change of a curve.
- To enable the students in applying the properties of tangent normal, curvature and asymptotes in tracing and reading the properties of the curves.
- To provide the students with deeper understanding of the techniques, principles and application of integral calculus.
- To orient the students to the nature of the envelope, derive its equation and apply them in describing family of curves.
- To familiarize the students in using integral calculus to evaluate the area of plane curves, lengths of arcs, and volumes and surfaces of the solids of revolution.
- To make the students able in writing the differential equation as an alternative form to the different types of the family of curves.
- To make the students able in applying differential equations to derive geometrical properties and to solve physical problems

### 3. Specific Objectives and Contents

Specific Objectives	Contents
<ul style="list-style-type: none"> <li>• Explain the limit and continuity in terms of <math>\epsilon</math>, <math>\delta</math>.</li> </ul>	<b>Unit I: Limits and Continuity (6)</b> 1.1 Use of $\epsilon - \delta$ in finding limit 1.2 Continuity and discontinuity 1.3 Geometrical meaning of continuity and discontinuity
<ul style="list-style-type: none"> <li>• Find the differential coefficient of different types of functions.</li> <li>• Explain the meaning successive differentiation.</li> <li>• Find the higher order derivatives of some specific functions.</li> <li>• State and prove Leibnitz theorem.</li> </ul>	<b>Unit II: Higher Order Derivatives (8)</b> 2.1 Differentiation of hyperbolic and step functions 2.2 Definitions and notations of higher order derivatives 2.3 nth order derivative of the functions as $x^m$ , $(ax+b)^m$ , $\sin(ax+b)$ , $\log(ax+b)$ etc. 2.4 Leibnitz theorem
<ul style="list-style-type: none"> <li>• Prove mean value theorems.</li> <li>• Interpret Rolles' theorem, Lagranges' mean value theorem</li> </ul>	<b>Unit III: Expansion of Functions (12)</b> 3.1 Rolle's Theorem 3.2 Lagrange's mean value theorem

<p>and Cauchy's mean value theorem.</p> <ul style="list-style-type: none"> <li>• Verify the mean value theorems for some functions.</li> <li>• Prove Taylor's theorem in finite and infinite forms.</li> <li>• Expand some functions in finite and infinite forms by using Maclaurin's series.</li> </ul>	<p>3.3 Canchy's mean value theorem  3.4 Taylors theorem with different form of remainders  3.5 Maclaurins' theorem  3.6 Verfication of Rolle's theorem, Lagrange's mean value theorem and Cauchy's mean value theorem  3.7 Expansion of functions <math>e^x</math>, <math>\sin x</math>, <math>\log x</math> etc. in finite and infinite forms using Maclaurin's series</p>
<ul style="list-style-type: none"> <li>• Prove and generalize L Hospital's theorem.</li> <li>• Find the limits of functions of different indeterminate forms.</li> </ul>	<p><b>Unit IV: Indeterminate forms (5)</b>  4.1 Examples of various indeterminate forms  4.2 L hospital's theorem (of <math>\div</math> form) and its generalization  4.3 Indeterminate forms: <math>\frac{\infty}{\infty}</math>, <math>\infty \times 0</math>, <math>\infty - \infty</math>, <math>1^\infty</math>, <math>1^\infty</math>, <math>\infty^\infty</math> (introduction with- examples)  4.4 Limits of functions of indeterminate forms</p>
<ul style="list-style-type: none"> <li>• Define partial derivatives w.r.t. x, y and z.</li> <li>• Interpret geometrically the partial derivatives of first order of two variables.</li> <li>• Find partial derivatives of higher order.</li> <li>• State and prove Euler's theorem on homogeneous functions and verify the theorem.</li> <li>• Explain total differentials.</li> <li>• Find <math>\frac{dy}{dx}</math> of implicit functions using partial derivatives.</li> </ul>	<p><b>Unit V: Partial differentiation (10)</b>  5.1 Limits and continuity of functions of two variables  5.2 Definition of partial derivatives and interpretation of first order  5.3 Partial derivatives of higher order  5.4 Homogeneous functions and Euler's theorem on two and three variables with its converse  5.5 Theorems on total differentials  5.6 Theorem on the derivative of composite functions  5.7 Differentiation of implicit functions</p>
<ul style="list-style-type: none"> <li>• Derive equation of tangent and normal of curves in explicit, implicit and parametric forms.</li> <li>• Find the angle of intersection of the curves in Cartesian and polar forms.</li> <li>• Find the length of tangent, normal, sub-tangent and subnormal in Cartesian and polar forms.</li> <li>• Find the derivative of arc length in Cartesian and polar forms.</li> <li>• Derive the angle between radius vector and tangent.</li> <li>• Find the length of perpendicular from pole on tangent.</li> <li>• Find pedal equation of the curves in Cartesian &amp; polar forms.</li> </ul>	<p><b>Unit VI: Tangent and Normal (10)</b>  6.1 Equation of tangent and normal  6.2 Problems on tangent and normal  6.3 Angle of intersection of the curves in Cartesian and polar forms  6.4 Length of tangent, normal, subtangent, subnormal in Cartesian and polar forms  6.5 Derivative of are length (Cartesian and polar form)  6.6 Angle between radius vector and tangent  6.7 Pedal equation of Cartesian and polar curves</p>
<ul style="list-style-type: none"> <li>• Define increasing and decreasing functions, concavity, convexity, point of inflection, stationary point, saddle point.</li> <li>• Derive necessary and sufficient conditions for maximum and</li> </ul>	<p><b>Unit VII: Maxima and Minima (10)</b>  7.1 Definitions of increasing and decreasing functions, concavity, convexity, point of inflection, stationary point, and saddle point  7.2 Conditions for concavity and convexity  7.3 Necessary and sufficient condition for</p>

<p>minimum.</p> <ul style="list-style-type: none"> <li>• Determine the conditions for maximum and minimum of the functions of two and three variables.</li> <li>• Solve the problems on maximum and minimum (application type).</li> </ul>	<p>maximum and minimum of functions of one, two or three variables.</p> <p>7.4 Extreme values under subsidiary conditions 7.5 Lagrange's method of undetermined multipliers 7.6 Problems on maxima and minima of two or three variables</p>
<ul style="list-style-type: none"> <li>• To give meaning of curvature</li> <li>• Find radius of curvature of different curves.</li> <li>• Find radius of curvature at origin</li> <li>• Deduce the chord of curvature through the origin (pole).</li> <li>• Define center of curvature, circle of curvature, evolutes, involutes.</li> <li>• Deduce the expressions for center of curvature.</li> </ul>	<p><b>Unit VIII: Curvature (10)</b></p> <p>8.1 Definition of curvature and its intuitive meaning 8.2 Radius of curvature of different types of curves 8.3 Curvature at origin 8.4 Chord of curvature through the origin (pole) 8.5 Center of curvature 8.6 Circle of curvature 8.7 Center of curvature and its property</p>
<ul style="list-style-type: none"> <li>• Define asymptotes.</li> <li>• Find asymptotes parallel to x-axis and y-axis.</li> <li>• Find oblique asymptotes.</li> <li>• Find asymptotes of curves in polar form.</li> </ul>	<p><b>Unit IX: Asymptotes (6)</b></p> <p>9.1 Definition of asymptotes with illustration in figure 9.2 Asymptotes parallel and non-parallel to the axes 9.3 Asymptotes of algebraic curves 9.4 Asymptotes of polar curves</p>
<ul style="list-style-type: none"> <li>• Describe rules for tracing curves in Cartesian and polar forms.</li> <li>• Trace some well-known curves in Cartesian and polar forms.</li> <li>• Define envelope.</li> </ul>	<p><b>Unit X: Curve Tracing (6)</b></p> <p>10.1 Rules for tracing Cartesian and polar curves 10.2 Tracing curves of some well known curves</p>
<ul style="list-style-type: none"> <li>• Give analytical definition of envelope of one parameter family of curves.</li> <li>• Determine envelope of one parameter family of curves.</li> <li>• Define two parameter family of curves.</li> <li>• Determine envelope of two parameter family of curves.</li> </ul>	<p><b>Unit XI: Envelope (6)</b></p> <p>11.1 Envelope and its examples 11.2 Envelope of straight lines 11.3 Envelope of a family of curves 11.4 Envelope of two parametric family of curves</p>
<ul style="list-style-type: none"> <li>• Integrate different types of functions of standard forms by different methods</li> </ul>	<p><b>Unit XII: Indefinite Integral (6)</b></p> <p>12.1 Integration of some standard integrals</p>
<ul style="list-style-type: none"> <li>• Define integration as the limit of a sum.</li> <li>• Give the geometrical interpretation of <math>\int_a^b f(x) dx</math>.</li> <li>• To state and prove the theorems and properties of definite-integral.</li> <li>• Solve the problems of definite integral by definition and using properties.</li> <li>• Find the integration of infinite (or improper) integrals.</li> </ul>	<p><b>Unit XIII: Definite integral (6)</b></p> <p>13.1 Integration as the limit of a sum 13.2 Geometrical interpretation of <math>\int_a^b f(x) dx</math> 13.3 General properties of definite integral 13.4 Methods of evaluating infinite (or improper) integrals</p>

<ul style="list-style-type: none"> <li>• Find the reduction formula for some standard integrals.</li> <li>• Define Beta and Gamma function.</li> <li>• Prove the properties of beta and gamma functions.</li> <li>• Apply the properties of Beta &amp; Gamma functions to evaluate some integrals.</li> </ul>	<p><b>Unit XIV: Reduction formulae, and Beta and Gamma functions (10)</b></p> <p>14.1 Reduction formulae for some special functions  14.2 Definition of Beta and Gamma functions  14.3 Properties of Beta and Gamma functions</p>
<ul style="list-style-type: none"> <li>• Find area of the curves in both Cartesian and polar forms.</li> <li>• Find the sectorial area of plane regions.</li> <li>• Find the length of arc of curve in both Cartesian and polar forms.</li> <li>• Find the intrinsic equation from Cartesian, Polar and Pedal equations.</li> <li>• Find the surface area and volume of solids of revolution: the axes of revolution being the x-axis, y-axis or any line in the plane.</li> </ul>	<p><b>Unit XV: Quadrature, Rectification, Volume and Surface Area of Revolution (16)</b></p> <p>15.1 Area in Cartesian coordinates  15.2 Area in polar coordinates  15.3 Area between two curves  15.4 Length of the arc of curve in Cartesian and polar form  15.5 Intrinsic equations from Cartesian and polar equations  15.6 Volume of solids of revolution  15.7 Surface area of solids of revolution (the axes being x-axis, y-axis or any line)</p>
<ul style="list-style-type: none"> <li>• Form the family of curves in terms of differential equation and interpret geometrically the meaning of differential equation.</li> <li>• Solve equation of the first order and first degree homogeneous linear equations.</li> <li>• Solve equations of first order but not of the first degree solvable for p, x or y.</li> <li>• Solve linear differential equations with constant coefficients.</li> <li>• Solve homogeneous linear equations.</li> </ul>	<p><b>Unit XVI: Differential Equations (15)</b></p> <p>16.1 Ordinary differential equation of first degree  16.1.1 Meaning Concept and Definitions  16.1.2 Concept of ordinary differentiation equation  16.1.3 General and particular solution  16.1.4 Change of variable  16.1.5 Homogeneous equations  16.1.6 Equations reducible to homogeneous equations  16.1.7 Linear differential equation  16.1.8 Equations reducible to linear form  16.1.9 Concepts and types of orthogonal and oblique trajectories  16.2 Linear differential equations with constant coefficients  16.2.2 Equation of the second order  16.2.2 Auxiliary equation and their roots, complimentary functions  16.2.3 Particular Integral  16.2.4 Methods of finding particular Integral</p>

*Note: The figures in the parentheses indicate the approximate periods for the respective units.*

#### 4. Instructional Techniques

Because of the theoretical nature of the course, teacher-centered instructional techniques will be mostly used in teaching learning process. The teacher will adopt the following methods/techniques.

##### 4.1 General Instructional Techniques

- Lecture
- Discussion
- Demonstration
- Problem solving

##### 4.1 Specific Instruction Techniques

#### 5. Evaluation

Students will be evaluated on the basis of the written classroom test in between and at the end of the academic session, the classroom participation, presentation of the reports and other practical activities. The scores obtained will be used only for the feedback purposes. The Office of the Controller of Examinations will conduct the annual examination at the end of year to evaluate students' performance. The types, number and marks of the subjective and objective questions will be as follows.

Types of questions	Total questions to be asked	Number of questions to be answered and marks allocated	Total marks
Group A: Multiple choice items	20 questions	20 x 1 mark	20
Group B: Short answer questions	8 with 3 'or' questions	8 x 7 marks	56
Group C: Long answer questions	2 with 1 'or' question	2 x 12 marks	24

#### 6. Recommended Books and References

##### Recommended Books

- Koirala, S. P., Pandey, U. N., Pahari, N. & Pokharel, P. (2008). *A textbook on integral calculus*. Kathmandu: Vidyanthi Prakashan (**For units XI to XV**)
- Koirala, S. P., Pandey, U.N. & Pahari, N. P. (2007). *A textbook on differential calculus (2<sup>nd</sup> ed.)*. Kathmandu: Vidyanthi Prakashan (**For units I to X**)
- Maskey, S.M. (2008). *Calculus*. Kathmandu: Ratna Pustak Bhandar (**For units I to X**)

##### References

- Das, B. C. & Mukerjee, B.N. (2007). *Differential calculus*. Calcutta: UN Dhur and Sons (Pvt.) Ltd. India.
- Das, B. C. & Mukerjee, B.N. (2007). *Integral calculus*. Calcutta: UN Dhur and Sons. (Pvt.) Ltd. India
- Ghosh, R.K. & Maity K.C. (2002). *An introduction to analysis – Differential calculus part II*, India: New Central Book Agency (Pvt.) Ltd.
- Ghosh, R.K., Maity, K.C. (1998). *An introduction to analysis - Differential calculus part I (9<sup>th</sup> ed.)* India: New Central Book Agency (Pvt.) Ltd.
- Narayan, S. (1998). *Differential calculus*. New Delhi: Shyam Lal Charitable Trust.
- Pant, G.D. & Shrestha, G.S. (2007). *Integral calculus (4th ed.)*. Kathmandu: Sunila Prakashan
- Thomas, G. B. & Finney, R. L. (2004). *Calculus (9th ed.)*. Delhi: Pearson Education.
- Upreti, K. N. (2007). *Differential calculus*. Kathmandu: